INDIAN STATISTICAL INSTITUTE, BANGALORE CENTRE
B.Math.Hons. II Year, First Semester, 2001-02
Statistics - I, Midsemestral exam, 26.09.01
Calculator and Statistical Tables may be used
Marks are shown to the left of each question

(9) 1. The number of eggs (X) laid by an insect can be assumed to have the Poisson distribution with unknown mean λ. Once laid, each egg has an unknown chance p of hatching, and the hatching of one egg is independent of the hatching of the others.
(a) What is the probability distribution of the number (Y) of hatched eggs of this insect?
(b) What is the probability distribution of the number (Z) of eggs of this insect which are not hatched?
(c) What is the conditional probability distribution of $X|Y = y$?

(6) 2. Let $X_1, X_2, \ldots, X_n$ be a random sample from the distribution with density
$f(x|\lambda) = \lambda \exp(-\lambda x)$, if $x > 0$, where $\lambda > 0$.
Let $X_{(1)} < X_{(2)} < \cdots < X_{(n)}$ be the order statistics, and define the spacings: $Y_1 = X_{(1)}$ and $Y_i = \left(X_{(i)} - X_{(i-1)}\right)$, $2 \le i \le n$. Find the joint distribution of $(Y_1, \ldots, Y_n)$.

(8) 3. Let $X_1, X_2, \ldots, X_n$ be a random sample from $N(\mu, \sigma^2)$ with $n > 5$.
Let $a_j = \frac{1}{\sqrt{n}}(-1)^{j+1}$, $1 \le j \le n$, and $b_j = 0.5$, $1 \le j \le 4$, $b_j = 0$ for $j \ge 5$.
(a) Find the joint distribution of $Y_1 = \sum_{j=1}^{n} a_j X_j$ and $Y_2 = \sum_{j=1}^{n} b_j X_j$.
(b) Find the conditional distribution of $Y_2 | Y_1 + Y_2 = y$.

(8) 4. Let $X_1, X_2, \ldots, X_n$ be a random sample from the distribution with density
$f(x|\lambda, \theta) = \lambda \exp(-\lambda(x - \theta))$, if $x > \theta$, where $\lambda > 0$.
Find the method of moments estimators of λ and θ.

(5) 5. Forty nine measurements are recorded to several decimal places. Each of these measurements is rounded off to the nearest integer. The sum of the original 49 numbers is approximated by the sum of these integers. If we assume that the round-off errors are independent $U(-\frac{1}{2}, \frac{1}{2})$, compute approximately the probability that the sum of the integers is within 2 units of the true sum.

(8) 6. Consider the linear model $y_i = \beta x_i + \epsilon_i$, $i = 1, \ldots, n$ for the data $(x_1, y_1), \ldots, (x_n, y_n)$. Suppose the ϵ_i have mean 0, variance $\sigma^2 > 0$, and they are uncorrelated.
(a) Find the least squares estimate for β.
(b) Find the expectation and mean square error of the estimate in (a) above assuming that the linear model is true.

(6) 7. Let $X_1, X_2, \ldots, X_n$ be a random sample from $U(0, \theta)$, where $\theta > 0$ is unknown.
(a) What is the probability distribution of $X_{(n)}$, the largest order statistic?
(b) Show that $X_{(n)}$ is a consistent estimator of θ.